

[4th Ph.D. Summer School on “Mathematical Modeling of Complex Systems”, Cultural Foundation “Kritiki Estia”, Athens](https://nlsconf.physics.uoc.gr/)

**An Introduction to Hypernetworks**

**Lesson 5 Hypernetworks in Global Systems Science**

**Homework to be submitted by 23:00 Friday 22nd August 2014**

This is the final homework for the course. Students who have completed all the homework assignments can request a certificate.

Each question carries 20 marks.

**Question 1**

(a ) What is the ‘Representation Problem’ of Complex Systems? (See Slide 5 of Lesson 5, Lecture 2, <http://www.hypernetworks.info/HN-Athens-2B.pdf>).

(b ) How could the position of any human being be represented in space and time?

(c ) Could there be a formula that can predict where every human being will be in ten days?

(d ) Give two ways to measure the distance between two people in space and time?

**Question 2**

(a ) In Global System Science, what is meant by ‘System’?

(b ) In Global System Science, what is meant by ‘Global’?

(c ) Give five examples of Global Systems.

(d) Why do we need a ‘Global System Science’?

**Question 3**

Ebola, a highly infectious virus that can kill up to 90 percent of the people who catch it, has caused about a thousand deaths recently in Western Africa.

(a) Say why, or why not, Ebola is a Global System.

(b) Is Ebola a multilevel system? Illustrate your answer by giving examples at the microlevel of individual people, and the macrolevel of nations.

(c) The risk of Ebola spreading to Europe is considered to be low. How does this compare to the spread of flu?

(d) According to the Guardian newspaper (9th August 2014), Guinea has closed its borders with Sierra Leone and Liberia to prevent infected people crossing into Guinea. Will this protect the people of Guinea from Ebola? (http://www.theguardian.com/society/2014/aug/09/ebola-guinea-sierra-leone-liberia).

**Question 4**

Consider a bipartite relation, R, between the villages {v, v’, v”} and the set of people {a, b, … , m, n }. Person x is R-related to village y if person x lives in village y, or regularly visits it.

Let σ(v) = <a, b, c, d, e, f; R>, σ(v’) = <e, f, g, h, i, j; R>, and σ(v”) = <i, j, k, l, m, n; R>.

As shown in Figure 1, σ(v) shares two people with σ(v’), σ(v’) shares two people with σ(v”), but σ(v) shares no people with σ(v”).

h

g

b

σ(v”)

n

i

e

σ(v)

a

m

j

f

l

k

d

c

Figure 1. The connectivity between the villages.

(a ) Ebola is transmitted through chains of contact between infected people. For example, in Figure 1 a can infect e and e can infect i and i can infect m. This chain, a–e–i–m, has three links. How many chains, or ‘infection paths’ are there with three links between a and m? List them all.

(b) Let p be the probability of a person with Ebola infecting another person in the same village. Then the probability that a infects m along the path a–e–i–m is p\*p\*p = p3.

What is the probability that a do*es not* infect m along the path a–e–i–m?

(c) What is the probability that a *does not* infect m along any of the infection paths of length 3.

(d) What is the probability that a *does* infect m along at least one of the four infection paths?

**Question 5**

In Figure 2, let M = | σ(v) ∩ σ(v’) | be the number of shared vertices between σ(v) and σ(v’) and N = | σ(v’) ∩ σ(v”) | be the number of shared vertices between σ(v’) and σ(v”).

σ(v) σ(v’) σ(v”)

y1

y2

…

yN

x1

x2

…

xM

w z

 Figure 2

(a) How many infection paths of length three are there between w and z in Figure 2.

(b) What is the probability that w *does not* infect z along any of these paths?

 What is the probability that w *does* infect z along at least one of the paths.

(c) What is the difference between your answer and M\*N \* p3 when M = N = 10 and p = 0.1 ?

(d) Make an argument that the probability of w infecting z increases by a factor of q2 or more when σ(v) and σ(v”) are q-connected.