

[4th Ph.D. summer School on “Mathematical Modeling of Complex Systems”, Cultural Foundation “Kritiki Estia”, Athens](https://nlsconf.physics.uoc.gr/)

**An Introduction to Hypernetworks**

**Lesson 1 Sets, relations, and the Galois hypergraphs**

**Homework to be submitted by 23:00 Tuesday 30th June 2014**

a b c d e f

1 1 0 1 1 0

0 1 1 1 0 1

1 1 0 0 1 1

0 0 1 1 1 1

1 1 1 0 1 0

1 0 1 1 0 1

A

B

C

D

E

F

**Figure 1**

**The matrix M of the relation *R***

In Figure 1 let Y = { a, b, c, d, e, f} be a set of blocks, and let X = { A, B, C, D, E, F} be a set of structures made from the blocks. Write *x* *R y* if structure *x* includes block *y*.

Question 1. (a) What is R(A) ∩ R(C) ∩ R(E) ?

(b) What is R(a) ∩ R(b) ∩ R(e) ?

Question 2. (a) What is R(B) ∩ R(D) ∩ R(F) ?

(b) What is R(c) ∩ R(d) ∩ R(f) ?

Question 3. (a) What is the matrix, M’, obtained by swapping columns c and e of M?

(b) What is the matrix, M”, obtained by swapping rows B and E in M’ .

Question 4. Referring to M”, explain why {A, C, E} ↔ { a, b, e} is a Galois pair.

Question 5. Referring to M”, what is the other 3 x 3 Galois pair in the system?