Hypernetworks and Multilevel Systems

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http://www.hypernetworks.info/introductiontohypernetworks.html

Hypernetworks and Multilevel Systems

Available on the website

09-6-2014 Lesson 0 Introduction
23-6-2014 Lesson 1 Sets, relations, and the Galois hypergraphs
30-6-2014 Lesson 2 Simplicial Complexes and Q-analysis
07-7-2014 Lesson 3 Hypernetworks

Now

15-7-2014 Lesson 4 Hypernetworks and Multilevel Systems

Next Wednesday

23-7-2014 Lesson 5 Hypernetworks in Global Systems Science

http://www.hypernetworks.info/introductiontohypernetworks.html

Hypernetworks and Multilevel Systems

Mathematical Modelling of Complex Systems

Can mathematics represent *everything* ?

How?

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Relationships – e.g. networks, …

Numbers – e.g. analysis, calculus, statistics, …

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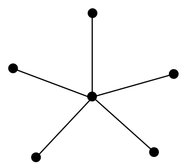


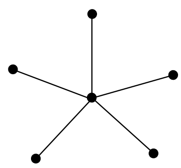
Network of similar people

e.g. `binge drinkers'

e.g. `obese people'

e.g. `computer games players'

People's networks - often people like themself



Network of similar people

e.g. \`binge drinkers'

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People's networks - often people like themself

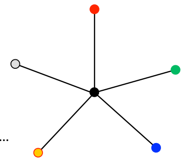
Network of heterogeneous people

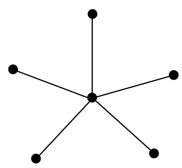
e.g. Family = Mum, Dad, Son, Daughter, Dog

e.g. Workmates = Designer, Engineer, Secretary

e.g. Team = Goalkeeper, Defender, Striker, Winger, …

People's networks often have people ***not*** like themselves





Network have structures & statistics

e.g. node degree distributions
long tailed versus Gaussian

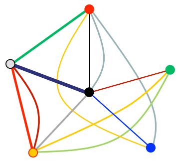
e.g. path lengths
shortest paths, network diameter

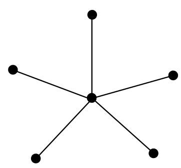
e.g. long links, small worlds, etc.

But often the details are important

Vertices are heterogeneous

Relationships are heterogeneous





Network have structures & statistics

e.g. node degree distributions
long tailed versus Gaussian

e.g. path lengths
shortest paths, network diameter

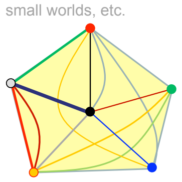
e.g. long links, small worlds, etc.

But often the details are important

Vertices are heterogeneous

Relationships are heterogeneous

… and may involve more than 2 vertices



What can networks do when there are relations between more than two things?

Hypergraphs !

$V =$

	E_1	E_2	E_3	E_4	E_5	E_6
x_1	0	0	0	0	1	0
x_2	0	0	0	1	1	0
x_3	1	0	0	1	0	0
x_4	1	0	0	0	0	0
x_5	1	1	0	0	0	0
x_6	0	0	1	0	0	0
x_7	0	0	1	1	0	1
x_8	0	1	1	0	0	0

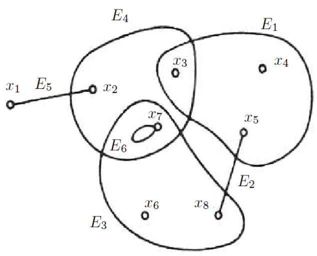
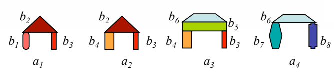


Fig. 1.1 The Berge hypergraph

Hypergraphs allow edges to have many vertices

Hypergraphs

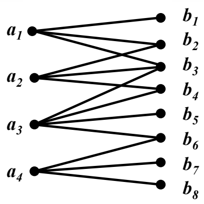
R	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8
a_1	1	1	1	0	0	0	0	0
a_2	0	1	1	1	0	0	0	0
a_3	0	0	1	1	1	1	0	0
a_4	0	0	0	0	0	1	1	1

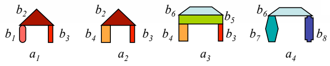


Hypergraphs

Bipartite Network

R	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8
a_1	1	1	1	0	0	0	0	0
a_2	0	1	1	1	0	0	0	0
a_3	0	0	1	1	1	1	0	0
a_4	0	0	0	0	0	1	1	1





Hypergraphs

Hypergraph edges

R	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8
a_1	1	1	1	0	0	0	0	0
a_2	0	1	1	1	0	0	0	0
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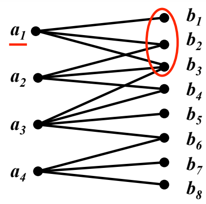
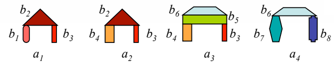
a_4	0	0	0	0	0	1	1	1





Hypergraphs

Hypergraph edges

R	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8
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a_2	0	1	1	1	0	0	0	0
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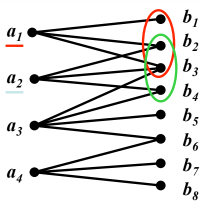

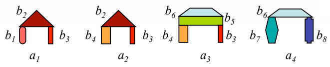

Hypergraphs

Hypergraph edges

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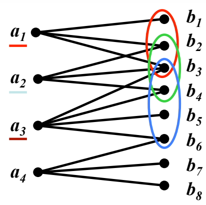

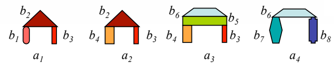

Hypergraphs

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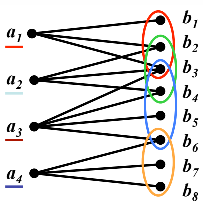
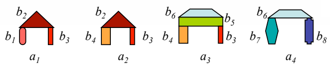
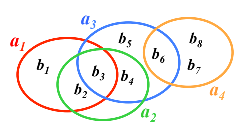




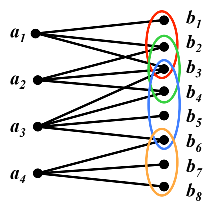
Hypergraphs

Hypergraph edges

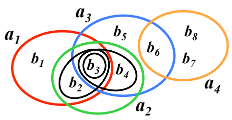


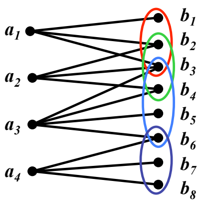
A ***hypergraph*** is a generalization of a graph, where edges may have many vertices.

The ***Galois hypergraph*** has all intersections.



Hypergraphs





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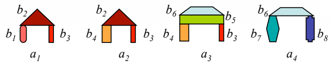
The ***Galois hypergraph*** has all intersections.

Hypergraphs

Blocks of related objects form *Galois pairs*

R	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8
a_1	1	1	1	0	0	0	0	0
a_2	0	1	1	1	0	0	0	0
a_3	0	0	1	1	1	1	0	0
a_4	0	0	0	0	0	1	1	1

$\{a_1, a_2\} \leftrightarrow \{b_2, b_3\}$



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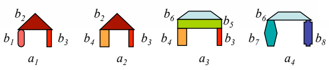
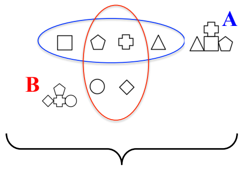
a_2	0	1	1	1	0	0	0	0
a_3	0	0	1	1	1	1	0	0
a_4	0	0	0	0	0	1	1	1

$\{a_2, a_3\} \leftrightarrow \{b_3, b_4\}$



Finding Maximal rectangles

	a	b	c	d	e	f
A	1	1	0	1	1	0
B	0	1	1	1	0	1
C	1	1	0	0	1	1
D	0	0	1	1	1	1
E	1	1	1	0	1	0
F	1	0	1	1	0	1



A and B share two blocks

Finding Maximal rectangles

	a	b	c	d	e	f
A	1	1	0	1	1	0
B	0	1	1	1	0	1
C	1	1	0	0	1	1
D	0	0	1	1	1	1
E	1	1	1	0	1	0
F	1	0	1	1	0	1

	a	b	e	d	c	f
A	1	1	1	1	0	0
B	0	1	0	1	1	1
C	1	1	1	0	0	1
D	0	0	1	1	1	1
E	1	1	1	0	1	0
F	1	0	0	1	1	1

Swap columns c and e

Finding Maximal rectangles

	a	b	c	d	e	f
A	1	1	0	1	1	0
B	0	1	1	1	0	1
C	1	1	0	0	1	1
D	0	0	1	1	1	1
E	1	1	1	0	1	0
F	1	0	1	1	0	1

	a	b	e	d	c	f
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D	0	0	1	1	1	1
E	1	1	1	0	1	0
F	1	0	0	1	1	1

Swap rows B and E

	a	b	e	d	c	f
A	1	1	1	1	0	0
E	1	1	1	0	1	0
C	1	1	1	0	0	1
D	0	0	1	1	1	1
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Finding Maximal rectangles

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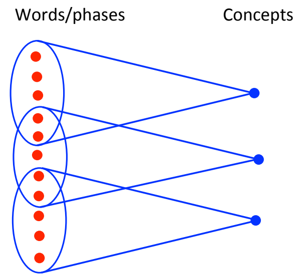
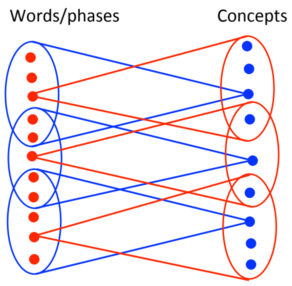
B	0	1	0	1	1	1
F	1	0	0	1	1	1

Galois Pairs - Example

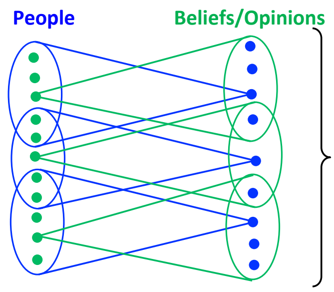


Galois Pairs - Example



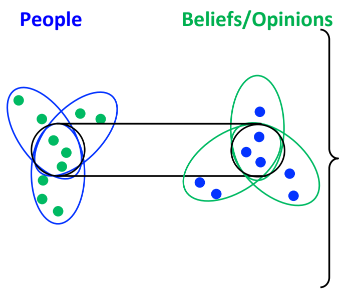
Words, concepts and contexts are intertwined Galois structures

Galois Pairs - Example



Words, concepts and contexts are intertwined Galois structures

Galois Pairs - Example

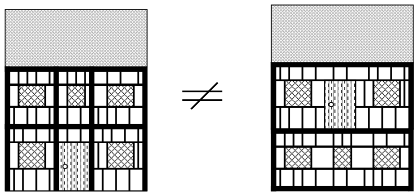


Words, concepts and contexts are intertwined Galois structures

Hypergraphs are beautiful structures, but ..

Hypergraphs are beautiful structures, but ..

they are set theoretic & not rich enough



Same set of parts but arranged differently

Hypergraphs and beautiful structure, but ..

they are set theoretic & not rich enough

{ D, O, G } = { G, O, D }   !

Hypergraphs and beautiful structure, but ..

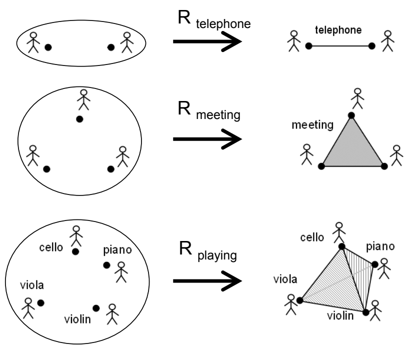
they are set theoretic & not rich enough

{ D, O, G } = { G, O, D }   !

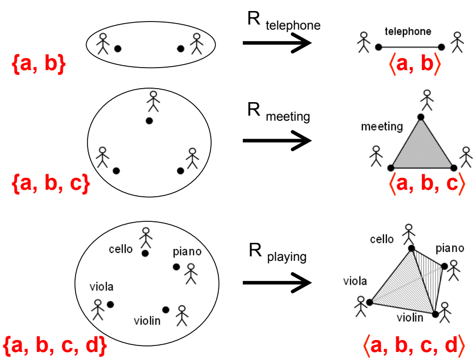
Simplices are better:

< D, O, G > != < G, O, D>

From sets to simplices



From sets to simplices



Simplices generalise edges in networks

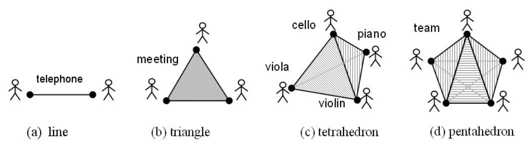
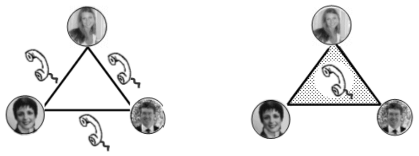


Figure 12. Representing relationships by multidimensional polyhedra

Binary relations are not rich enough



3 binary relations $\neq$ one 3-ary relation

<mother, father> + <mother, daughter> + <father, daughter>

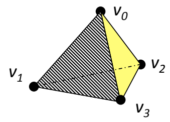
= / = < mother, father, daughter>

From networks to simplicial complexes

An abstract ***p-simplex*** is an ordered set of vertices,

$\sigma_p = \langle v_0, v_1, v_2, \dots, v_p \rangle$.

e.g. the tetrahedron



$\sigma_3 = \langle v_0, v_1, v_2, v_3 \rangle$.

From networks to simplicial complexes

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A *face* is a sub-simplex.

e.g. a triangle

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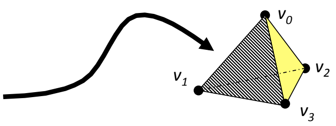

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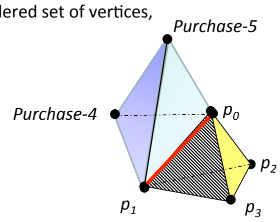

Example

shopping carts connected by

$\sigma = \langle p_0, p_1 \rangle$.

From networks to simplicial complexes

An abstract ***p-simplex*** is an ordered set of vertices,

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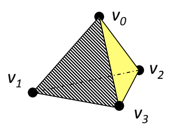
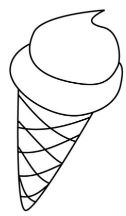


A *simplicial complex* is a set of simplices with all their faces

$\sigma_3 = \langle v_0, v_1, v_2, v_3 \rangle$.

From Networks to Hypernetworks



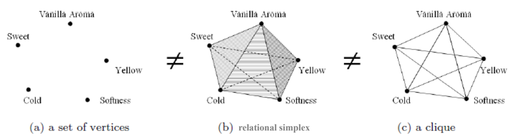
Gestalt Psychologist Katz:

Vanilla Ice Cream ≠ cold + yellow + soft + sweet + vanilla

it is a Gestalt – experienced as a whole

⟨ cold, yellow, soft, sweet, vanilla ⟩

From Networks to Hypernetworks

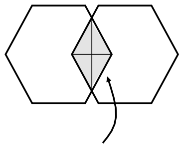


Set of vertices ≠ simplex ≠ clique

⟨ cold, yellow, soft, sweet, vanilla ⟩

From networks to simplicial complexes

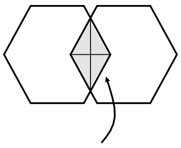
Interesting structures



q-near

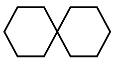
From networks to simplicial complexes

Interesting structures

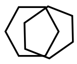


q-near

$$\text{eccentricity}(\sigma) = \frac{|\,\sigma\,| - |\,\sigma \cap \sigma'\,|}{|\,\sigma\,|}$$



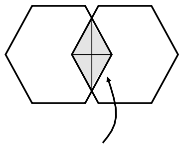
high eccentricity



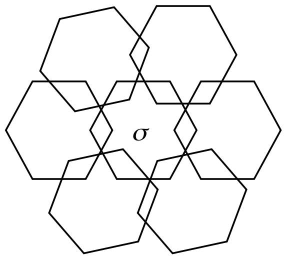
low eccentricity

From networks to simplicial complexes

Interesting structures

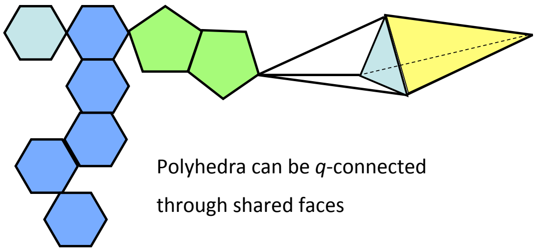


q-near



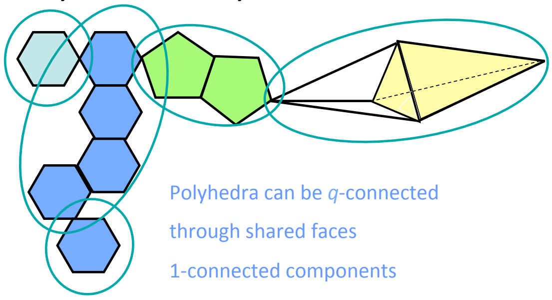
q-neighbourhood of σ

Polyhedral Connectivity



Polyhedra can be *q*-connected
through shared faces

Polyhedral Connectivity

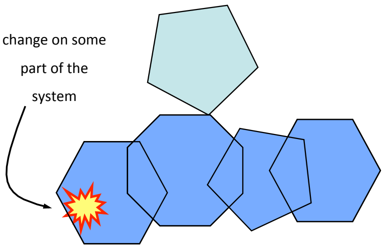


Polyhedra can be *q*-connected
through shared faces
1-connected components

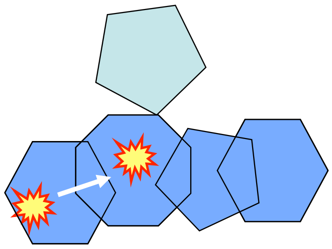
Q-analysis: listing q-components

Polyhedral Connectivity & q-transmission

(q-percolation)

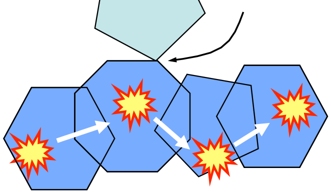


Polyhedral Connectivity & q-transmission



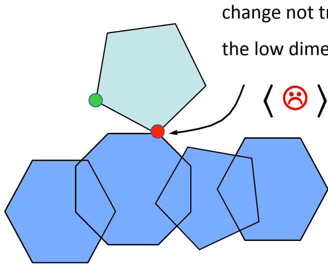
Polyhedral Connectivity & q-transmission

change not transmitted by
the low dimensional face



Polyhedral Connectivity & q-transmission

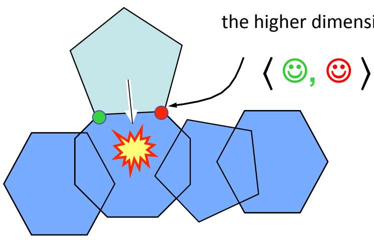
change not transmitted by
the low dimensional face



E.g. simplices are committees – one person ignored

Polyhedral Connectivity & q-transmission

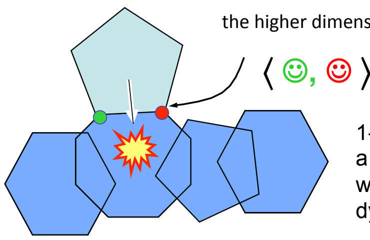
change *is* transmitted by
the higher dimensional face



E.g. simplices are committees – two people stronger

Polyhedral Connectivity & q-transmission

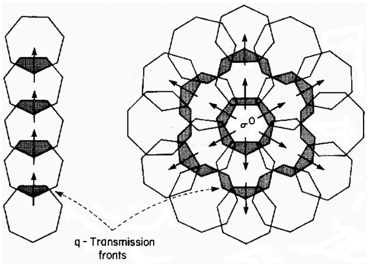
change *is* transmitted by
the higher dimensional face



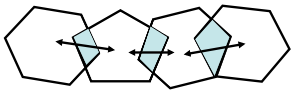
1-simplex is a ***structure*** with its own dynamics

E.g. simplices are committees – two people stronger

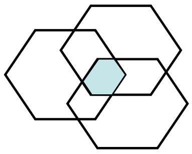
Polyhedral Connectivity & q-transmission



Intersections of simplices and dynamics

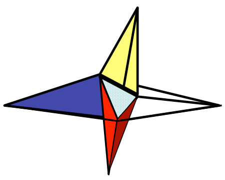


Shared faces are sites of interaction for ***pairs*** of simplices

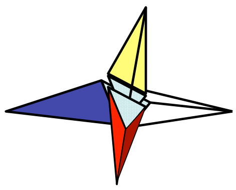


What about the intersection of more than two simplices?

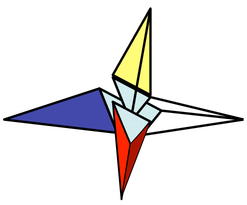
Intersections of simplices and dynamics



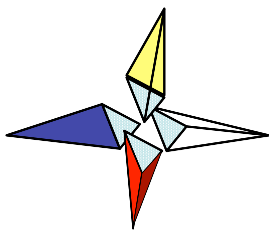
Intersections of simplices and dynamics



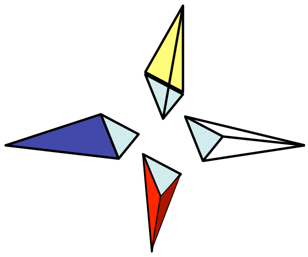
Intersections of simplices and dynamics



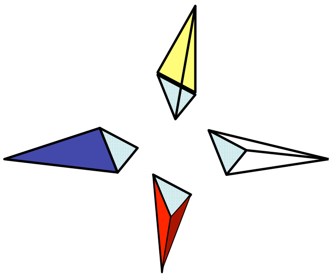
Intersections of simplices and dynamics



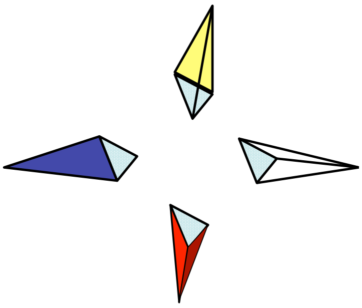
Intersections of simplices and dynamics



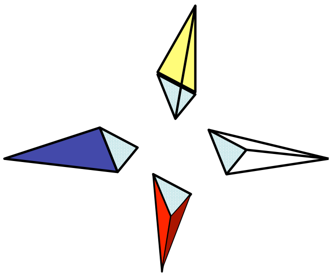
Intersections of simplices and dynamics



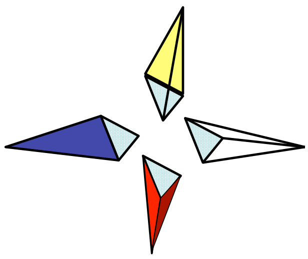
Intersections of simplices and dynamics



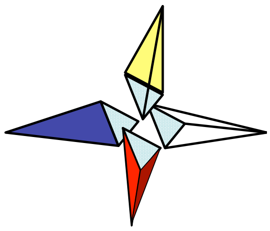
Intersections of simplices and dynamics



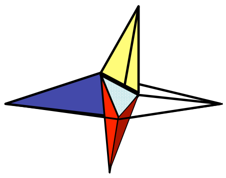
Intersections of simplices and dynamics



Intersections of simplices and dynamics

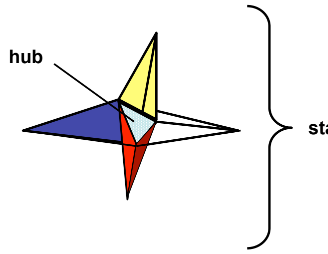


Intersections of simplices and dynamics



Intersections of simplices and dynamics

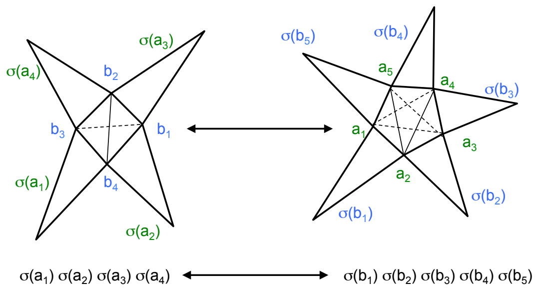
star-hub relationship is a **Galois connection**

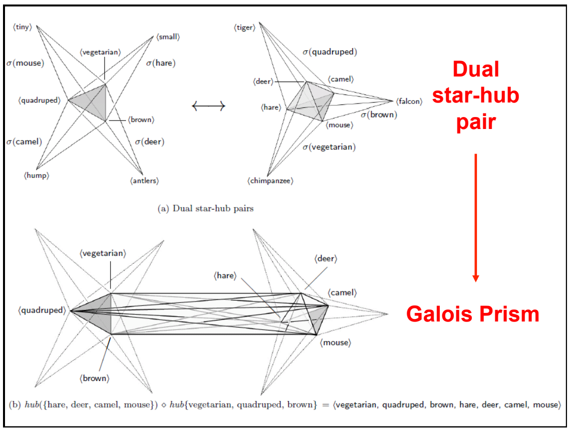


relational simplices have rich connectivity structures

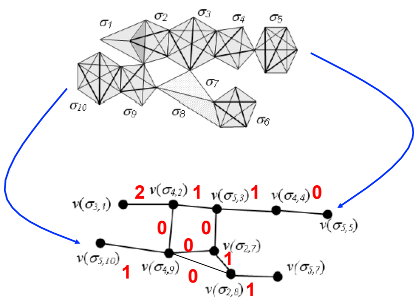
Intersections of simplices and dynamics

star-hub relationship is a **Galois connection**





q-graphs summarise the q-connectivity



q-graphs summarise the q-connectivity – **partly !**

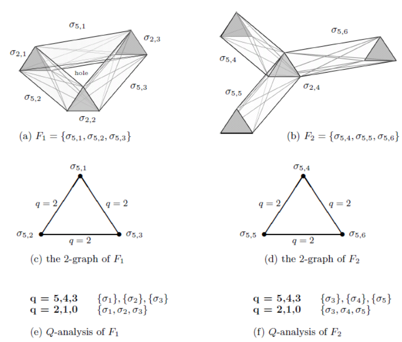
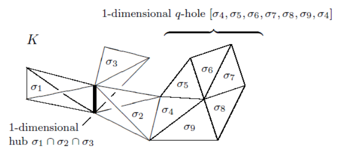


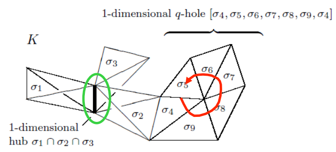
Fig. 4.29 Q-graphs cannot discriminate different topologies



(a) K has a 1-hub and a 1−hole

Fig. 5.42 The q-complex disambiguates hubs and holes

1-dimensional cycle

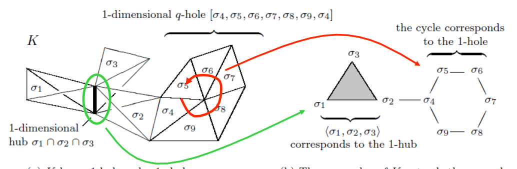


(a) K has a 1-hub and a 1−hole

Fig. 5.42 The q-complex disambiguates hubs and holes

1-dimensional hub ≠ cycle

1-dimensional cycle



(a) K has a 1-hub and a 1−hole (b) The q-complex of K extends the q-graph

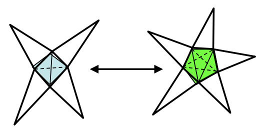
Fig. 5.42 The q-complex disambiguates hubs and holes

1-dimensional hub ≠ cycle

q-graph necessarily becomes the q-complex !

star-hub relationship is a **Galois connection**

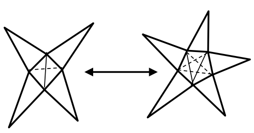
		b_1	b_2	b_3	b_4	b_5	
…	. . .	.	.	.	.	.	. . .
a_1	. . .	1	1	1	1	1	. . .
a_2	. . .	1	1	1	1	1	. . .
a_3	. . .	1	1	1	1	1	. . .
a_4	. . .	1	1	1	1	1	. . .
…	. . .	.	.	.	.	.	. . .



… but there is a combinatorial explosion of pairs

star-hub relationship is a **Galois connection**

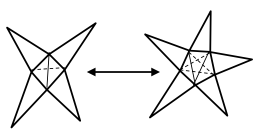
		b_1	b_2	b_3	b_4	b_5	
…	. . .	.	.	.	.	.	. . .
a_1	. . .	1	1	1	1	1	. . .
a_2	. . .	1	0	1	1	1	. . .
a_3	. . .	1	1	1	0	1	. . .
a_4	. . .	1	1	1	1	1	. . .
…	. . .	.	.	.	.	.	. . .



q-connected components - looser clusters of simplices

star-hub relationship is a **Galois connection**

		b_1	b_2	b_3	b_4	b_5	
…	. . .	.	.	.	.	.	. . .
a_1	. . .	1	1	1	1	1	. . .
a_2	. . .	1	0	1	1	1	. . .
a_3	. . .	1	1	1	0	1	. . .
a_4	. . .	1	1	1	1	1	. . .
…	. . .	.	.	.	.	.	. . .

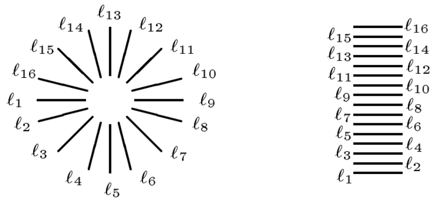


q-connected components - looser clusters of simplices

q-connected components form clusters of Galois pairs

… may make the component more tractable

From Complexes to Hypernetworks



(a) The sun illusion $\sigma_1 = \langle \ell_1, ..., \ell_{16}; R_1 \rangle$

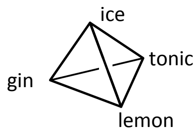
(b) the rectangle illusion $\sigma_2 = \langle \ell_1, ..., \ell_{16}; R_2 \rangle$

Simplices are not rich enough to discriminate things

Same parts, different relation, different structure & emergence

We must have *relational simplices*

From Networks to Hypernetworks



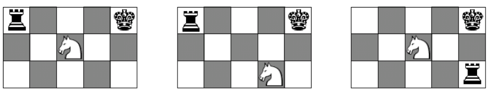
Gin & Tonic is a Gestalt !

Relational Simplex

⟨gin, tonic, ice, lemon; R_{mixed}⟩

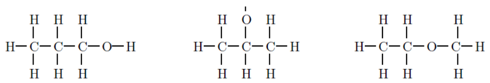
Another example of a relational simplex (or hypersimplex)

From Networks to Hypernetworks



(a) ⟨rook, knight, king; R_1⟩ (b) ⟨rook, knight, king; R_2⟩ (c) ⟨rook, knight, king; R_3⟩

Fig. 6.5 The knight fork in chess



(a) *n*-propyl alcohol (b) isopropyl alcohol (c) methyl-ethyl-ether

Fig. 6.6 Chemical isomers as relational simplices

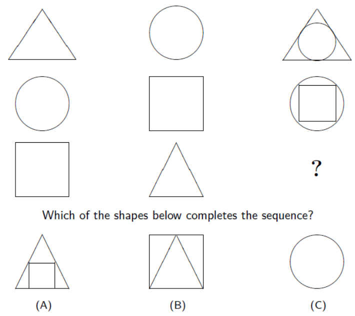


Fig. 6.8 An IQ test question

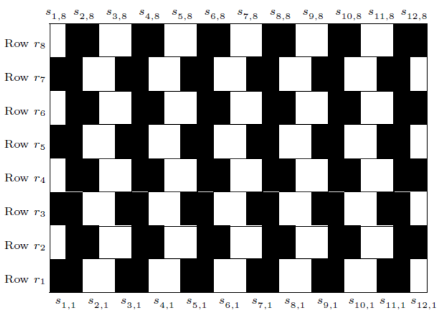


Fig. 6.11 The café wall illusion

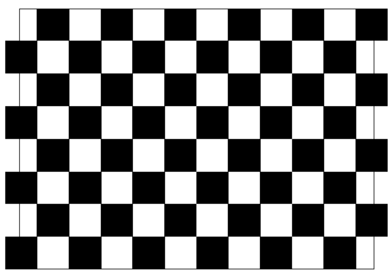


Fig. 6.12 The café wall illusion disappears for normal tilings

From Networks to Hypernetworks

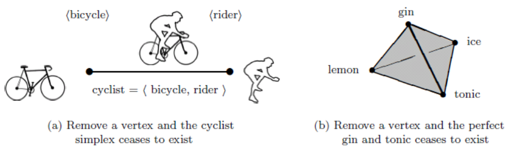


Fig. 4.8 Remove a vertex and the simplex ceases to exist.

From Networks to Hypernetworks

Definition

A ***hypernetwork*** is a set of hypersimplices

e.g.

⟨ cold + yellow + soft + sweet + vanilla; $R_{Vanilla\_Ice\_Cream}$ ⟩

The sun illusion
$\sigma_1 = \langle \ell_1, ..., \ell_{16}; R_1 \rangle$

the rectangle illusion
$\sigma_2 = \langle \ell_1, ..., \ell_{16}; R_2 \rangle$

Hypernetworks are another piece in the network jigsaw

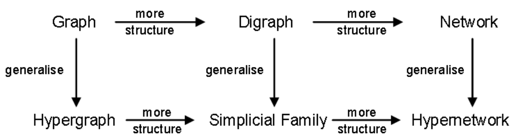


Fig. 4. The relationship between graphs, network, hypergraph and hypernetworks

Hypersimplices and Multilevel Systems

5.1 Systems of Systems of Systems

Most systems are characterised by having many subsystems and levels of description. They are made up of inextricably entangled multilevel social and physical subsystems with intra-level and inter-level bottom-up and top-top-down dynamics. They are *systems of systems*. In fact they are systems of systems of systems, and more generally multiple levels of systems of systems.

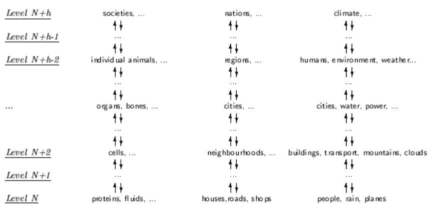
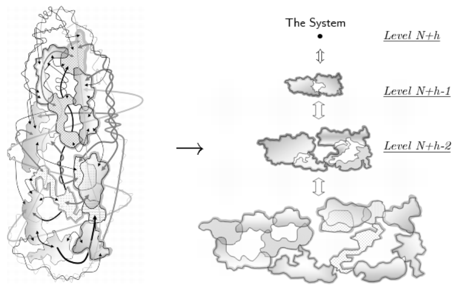


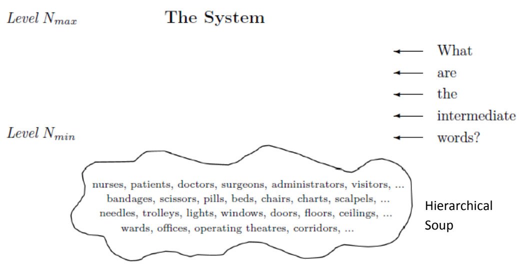
Fig. 5.1 Systems of systems of systems of systems ...

Multilevel Systems



Can highly entangled multilevel systems separated into well-defined levels ?

Multilevel Systems



The Intermediate Word Problem

Multilevel Systems

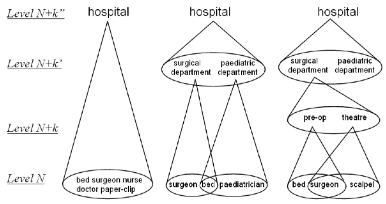
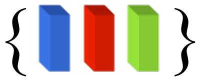


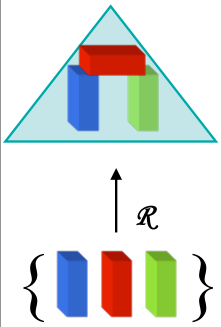
Fig. 5.4 Intermediate Words for a Hospital System

Formation of simplices ⇒ hierarchical structure

e.g. take a set of 3 blocks

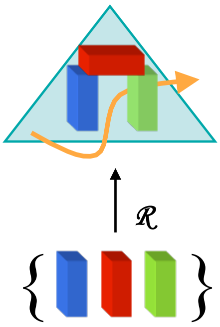


Formation of simplices ⇒ hierarchical structure



e.g. take a set of 3 blocks assembled by a 3-ary relation $\mathcal{R}$

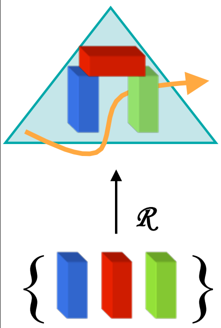
Formation of simplices ⇒ hierarchical structure



e.g. take a set of 3 blocks assembled by a 3-ary relation $\mathcal{R}$

The structure has an emergent property

Formation of simplices ⇒ hierarchical structure

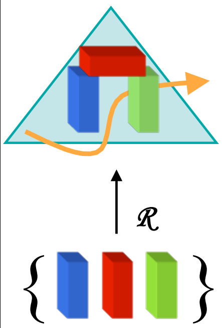


n-ary relation assembles elements into named structures at a higher level

Level N+1

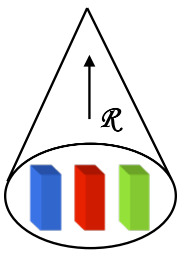
Level N

Formation of simplices ⇒ hierarchical structure



n-ary relation assembles elements into named structures at a higher level

Arch



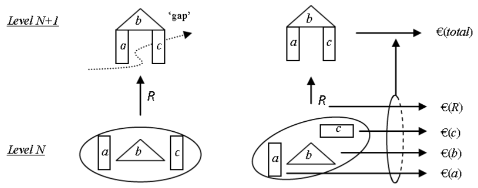


Fig 5. Aggregating *Level N* parts to *Level N+1* wholes, and aggregating numbers

AND and OR aggregations in multilevel systems

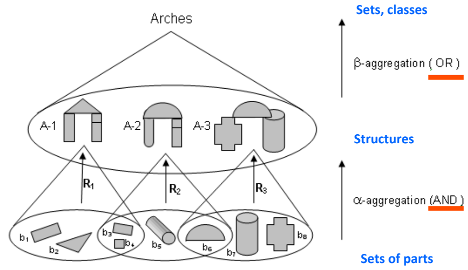


Figure 8. Two different types of multilevel aggregation

Conventional classification trees don't have alpha aggregations

Observing multilevel systems of systems of systems

Hypothesis 1

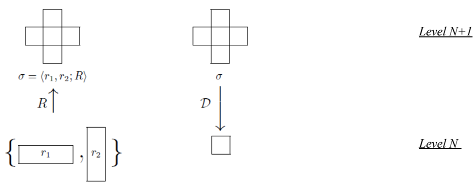
When we look at systems we see the whole & the parts

Hypothesis 2

Our brains create new multilevel structures

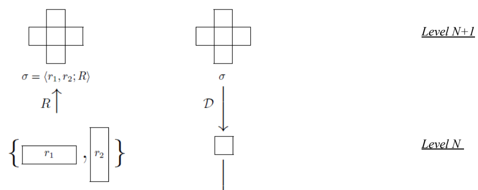
Aggregation – deconstruction downward dynamics in representing systems

Aggregation – deconstruction downward dynamics in representing systems



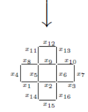
Create a new object at Level N !

Aggregation – deconstruction downward dynamics in representing systems

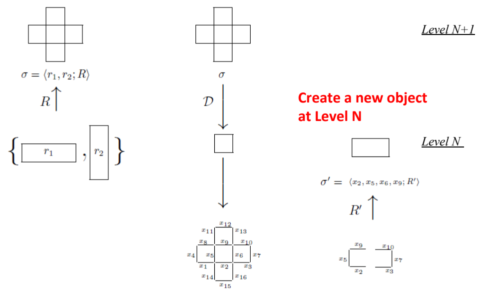


Create a new level - Level N-1 !

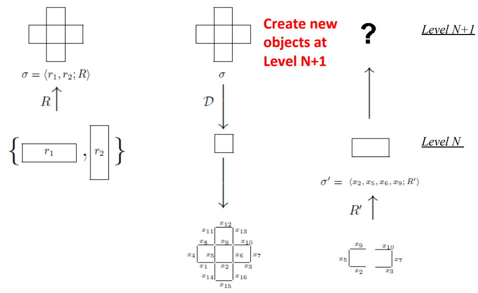
And new objects at this level



Aggregation – deconstruction downward dynamics in representing systems



Aggregation – deconstruction downward dynamics in representing systems



Applications

Digital texts are multilevel systems of hypersimplices

Text ⟷ 'meaning'
Paragraphs ⟷ 'meaning'
sentences ⟷ 'meaning'
phrases ⟷ 'meaning'
words ⟷ 'meaning'

Galois Pairs
texts ⟷ meanings

words ⟷ meanings
Galois pairs

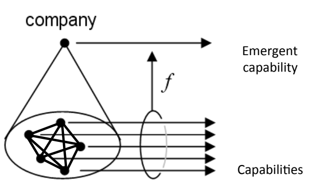
Backcloth and traffic

Relational simplices support patterns of numbers across their faces representing dynamical aspects of the systems.

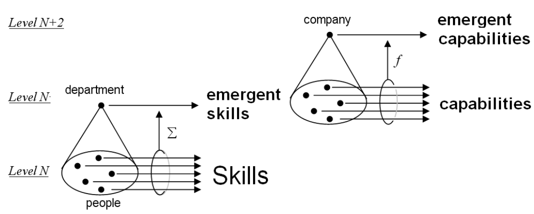
The simplices form a backcloth for the more dynamic traffic

… but there are also backcloth dynamics as relational simplices are formed.

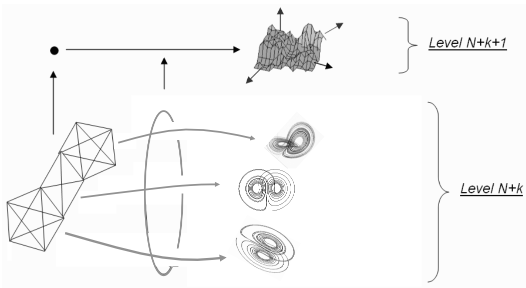
Multilevel patterns of numbers on the structure



Multilevel patterns of numbers on the structure



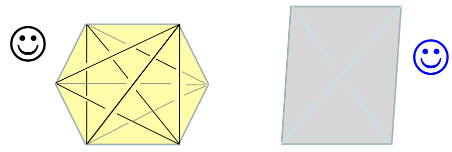
Dynamics on the hypernetwork backcloth



System dynamics as traffic on a fixed multilevel backcloth

Dynamics on the hypernetwork backcloth

opinions

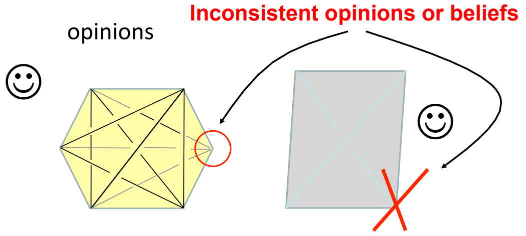


Person 1 Person 2

Dynamics on the hypernetwork backcloth

Inconsistent opinions or beliefs

opinions

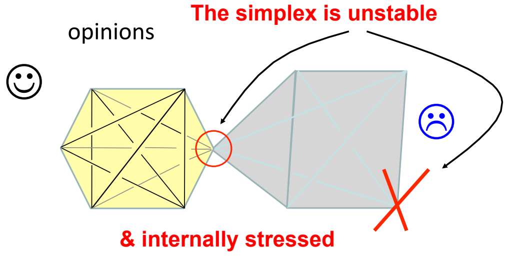


Person 1 Person 2

Dynamics on the hypernetwork backcloth

The simplex is unstable

opinions



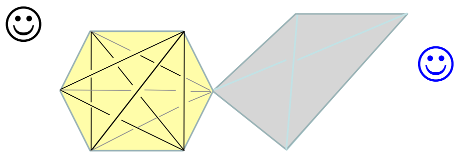
& internally stressed

Person 1 Person 2

Dynamics on the hypernetwork backcloth

The simplex becomes stable

opinions



& internally un-stressed

Person 1 Person 2

Dynamics

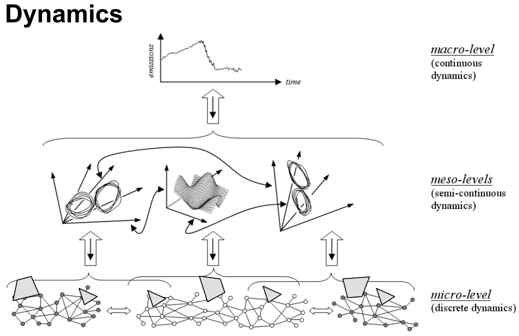


Figure 2. discrete micro-dynamic, semi-continuous meso-dynamics and continuous macro-dynamic

End of Lesson 1

Conclusions

Need a way of representing n-ary relations

Hypergraphs a first step, but not rich enough

Simplicial Complexes are better, but still not rich enough

Hypernetworks complete the relational jigsaw

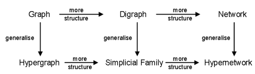


Fig. 4. The relationship between graphs, network, hypergraph and hypernetworks

Hypernetworks can represent multilevel systems

Necessary (if not sufficient) for complex systems